at a frequency of 1-2 Hz and amplitude of 1-2% did not induce any appreciable intensification of heat transfer in the investigated regimes associated with the ascending flow of supercritical helium in a vertical tube.

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THE FINAL STAGE OF DEGENERATION IN THE TURBULENT PATTERN FOR A PASSIVE TRACE COMPONENT IN A WAKE

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A multiparameter differential model has been used to derive asymptotic formulas for the behavior of a passive component in the final stage of degeneration in a turbulent wake.

Recently, considerable experience has been accumulated in calculating the characteristics of turbulent shear flows on the basis of multiparameter $\overline{u_i u_j}-\varepsilon_u$ differential models [1]. The advances in this area have provided a stimulus to constructing differential models for the transport of a passive component, which one can take as being the temperature if there is a slight temperature rise and buoyancy effects are negligible.

The passive scalar is a transportable substance , so that in most free flows the turbulent Peclet number P_λ varies along with R_λ from $P_\lambda \gg 1$ in the near region to P_λ in the far one. A study has been made [2] of the features in the final stage of degeneration in the pattern for a passive component by applying a Fourier transformation to the Navier–Stokes equations and then expanding the Fourier transforms as series, taking the first few terms in the expansion. The multiparameter differential model has been proposed [3, 4] to describe the scalar field.

A distinctive feature is that it contains functions of the turbulent Reynolds and Peclet numbers instead of the traditional empirical constants and incorporates the evolution of the scalar field as a function of R_λ and P_λ. In the zones of strong turbulence ($R_\lambda \gg 1$, $P_\lambda \gg 1$) and weak turbulence ($R_\lambda < 1$, $P_\lambda < 1$), one can replace the empirical functions by constants, which determine the damping of the model characteristics of the wake in the asymptotic cases R_λ, $P_\lambda \to \infty$ and R_λ, $P_\lambda \to 0$.

The following is the closed system of equations in a Cartesian coordinate system [4]:

$$\frac{D\overline{T}}{D\tau} = \varkappa \frac{\partial^2 \overline{T}}{\partial x_k^2} - \frac{\partial \overline{u_k t}}{\partial x_k},$$

$$\frac{D\overline{u_i t}}{D\tau} = \frac{\partial}{\partial x_k}\left[\alpha_{ut}\frac{q^2}{\varepsilon_u}\left(\overline{u_i u_l}\frac{\partial \overline{u_k t}}{\partial x_l} + \overline{u_k u_l}\frac{\partial \overline{u_i t}}{\partial x_l} + \overline{u_l t}\frac{\partial \overline{u_i u_k}}{\partial x_l}\right) + \frac{\nu+\varkappa}{2}\frac{\partial \overline{u_i t}}{\partial x_k}\right] - b_{ut}\overline{u_k t}\frac{\partial \overline{U}_i}{\partial x_k} - \overline{u_i u_k}\frac{\partial \overline{T}}{\partial x_k} - c_{ut}\frac{\varepsilon_t}{\overline{t^2}}\overline{u_i t},$$

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$$\frac{D\overline{t^2}}{D\tau}=\frac{\partial}{\partial x_k}\left[\alpha_{tt}\frac{\overline{t^2}}{\varepsilon_t}\left(\overline{u_k u_l}\frac{\partial\overline{t^2}}{\partial x_l}+2\overline{u_l t}\frac{\partial \overline{u_k t}}{\partial x_l}\right)+\varkappa\frac{\partial\overline{t^2}}{\partial x_k}\right]-2\overline{u_k t}\frac{\partial\overline{T}}{\partial x_k}-2\varepsilon_t,$$

$$\frac{D\varepsilon_t}{D\tau}=\frac{\partial}{\partial x_k}\left[\left(\varkappa+\alpha_{\varepsilon t}\frac{\overline{u_k u_l}\,\overline{t^2}}{\varepsilon_t}\right)\frac{\partial\varepsilon_t}{\partial x_k}\right]-b_{\varepsilon t}\frac{\overline{u_i t}\,\varepsilon_u}{q^2}\frac{\partial\overline{T}}{\partial x_i}-$$

$$-b_{\varepsilon u}\frac{\overline{u_i u_j}\,\varepsilon_t}{q^2}\frac{\partial\overline{U}_i}{\partial x_j}-F_{t1}\frac{\varepsilon_u\varepsilon_t}{q^2}-F_{t2}\frac{\varepsilon_t^2}{\overline{t^2}}.$$

Here α_{ut}, α_{tt}, $\alpha_{\varepsilon t}$, b_{ut} are empirical constants, while c_{ut}, $b_{\varepsilon t}$, $b_{\varepsilon u}$, F_{t1}, F_{t2} are empirical functions of the turbulent Reynolds and Peclet numbers and also in general of the parameter $R = \varepsilon_t q^2/\varepsilon_u \overline{t^2}$.

We introduce the following dimensionless quantities by taking the characteristic quantities as the diameter of a body of rotation or the transverse dimension of a planar body d, the speed of the incident U_∞, and the temperature T_∞ of this:

$$x=\frac{x_1}{d};\ r=\frac{x_2}{d};\ E=\frac{\overline{u_i u_i}}{U_\infty^2};\ D_u=\varepsilon_u\frac{d}{U_\infty^3};\ R_\infty=\frac{U_\infty d}{\nu};$$

$$P_\infty=\frac{U_\infty d}{\varkappa};\ T=\frac{\overline{T}-T_\infty}{T_\infty};\ \theta=\frac{\overline{t^2}}{T_\infty};$$

$$D_t=\varepsilon_t\frac{d}{U_\infty T_\infty^2};\ R_t=\frac{\overline{u_2 t}}{rU_\infty T_\infty}.$$

The inertial forces become negligible by comparison with the viscous ones for R_λ, $P_\lambda \to 0$ so the initial equation system and the conservation condition for the excess heat content I_t take the form

$$\frac{\partial T}{\partial x}=\frac{1}{P_\infty r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial T}{\partial r}\right);\quad I_t=\int_0^\infty r^n T(x,\ r)\,dr=\text{const},\tag{1}$$

$$\frac{\partial R_t}{\partial x}=\frac{1}{P_\infty r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial R_t}{\partial r}\right)+\frac{2}{rP_\infty}\frac{\partial R_t}{\partial r}-\frac{E}{3r}\frac{\partial T}{\partial r}-c_{ut}\frac{D_t}{\theta}R_t,\tag{2}$$

$$\frac{\partial\theta}{\partial x}=\frac{1}{P_\infty r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial\theta}{\partial r}\right)-2D_t,\tag{3}$$

$$\frac{\partial D_t}{\partial x}=\frac{1}{P_\infty r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial D_t}{\partial r}\right)-F_{t1}\frac{D_u D_t}{E}-F_{t2}\frac{D_t^2}{\theta}.\tag{4}$$

The value n = 0 corresponds to planar flow and n = 1 to axially symmetrical flow.

We examine the dependence of solution to (1)-(4) on c_{ut}, F_{t1}, F_{t2} on the assumption of a self-modeling flow in the final stage of degeneration, where we represent the parameters in (1)-(4) as

$$T(x,\ \eta)=T_0(x+x_0)^{nT}f_t(\eta),\ D_t(x,\ \eta)=D_{t0}(x+x_0)^{nDT}f_{Dt}(\eta),$$

$$R_t(x,\ \eta)=R_{t0}(x+x_0)^{nRT}f_{Rt}(\eta),\ E(x,\ \eta)=E_0(x+x_0)^{nE}f_E(\eta),$$

$$\theta(x,\ \eta)=\theta_0(x+x_0)^{n\theta}f_\theta(\eta),\ \eta=\frac{r}{\sqrt{x+x_0}}.$$

The exponents in the degeneration laws and the functions of the variable η satisfy ordinary differential equations:

$$nT\cdot f_t-\frac{\eta}{2}f_t'=\frac{1}{\eta^n P_\infty}(\eta^n f_t')',\tag{5}$$

$$nRT\cdot f_{Rt}-\frac{\eta}{2}f_{Rt}'=\frac{1}{\eta^n P_\infty}(\eta^n f_{Rt}')'+2\frac{f_{Rt}'}{\eta P_\infty}-\frac{T_0 E_0}{3\eta R_{t0}}f_E f_t'-c_{ut}\frac{D_{t0}}{\theta_0}\frac{f_{Dt}f_{Rt}}{f_\theta},\tag{6}$$

$$n\theta \cdot f_\theta - \frac{\eta}{2} f'_\theta = \frac{1}{\eta^n P_\infty} (\eta^n f'_\theta)' - 2 \frac{D_{t0}}{\theta_0} f_{Dt}, \tag{7}$$

$$nDT \cdot f_{Dt} - \frac{\eta}{2} f'_{Dt} = \frac{1}{\eta^n P_\infty} (\eta^n f'_{Dt})' - F_{t1} \frac{D_{u0} D_{t0}}{E_0} \frac{f_{Du} f_{Dt}}{f_E} - F_{t2} \frac{D_{t0}^2}{\theta_0} \frac{f_{Dt}}{f_\theta}, \tag{8}$$

together with the condition for conservation of the excess heat content

$$I_t = \int_0^\infty \eta^n f_t(\eta)\, d\eta = \text{const} \tag{9}$$

and the system of inequalities

$$nT > nRT + 1;\; n\theta > nT + nRT + 1;\; nDT > nRT + nT. \tag{10}$$

Each of the functions in (5)-(8) should satisfy the following boundary conditions:

$$f'(0) = 0;\; \lim_{\eta \to \infty} f(\eta) = \lim_{\eta \to \infty} f'(\eta) = 0. \tag{11}$$

It follows from [5] that the turbulent kinetic energy and the dissipation rate satisfy the following relations in the final stage in a wake (apart from a planar wake with nonzero excess momentum):

$$E(x, r) = E_0 (x + x_0)^{-\frac{n+6}{2}} \Phi(R_\infty),\; D_u(x, r) = \frac{5}{4} \frac{E(x, r)}{x + x_0}, \tag{12}$$

where

$$\Phi(z) = \exp\left(-\frac{z}{4} \frac{r^2}{x + x_0}\right).$$

We take (7) and (8) together to get

$$f_{Dt} = \frac{1 - 1{,}25 F_{t1}}{F_{t2} - 2} \frac{\theta_0}{D_{t0}} f_\theta;\; n\theta = -\frac{n+1}{2} - \frac{2 - 2{,}5 F_{t1}}{F_{t2} - 2} = nDT + 1, \tag{13}$$

i.e., the damping exponent for the temperature fluctuations is dependent on F_{t1} and F_{t2}. It follows from [4] that $\lim\limits_{R_\lambda, P_\lambda \to 0} F_{t1} = 0$, $\lim\limits_{R_\lambda, P_\lambda \to 0} F_{t2} = 10/3$, and in that case $n\theta = -(n + 4)/2$, which agrees with the results of [2]. Equation (7) is readily transformed to $2(\eta^n f'_\theta)' + P_\infty(\eta^{n+1} f_\theta)' = 0$ and has the solution $f_\theta = \Phi(P_\infty)$, so in the final stage $\theta(x, r) = \theta_0 (x + x_0)^{-(n+4)/2} \Phi(P_\infty)$, $D_t(x, r) = 3/4\, \theta(x, r)/(x + x_0)$, where the microscopic scale λ_t increases in accordance with the law $\sqrt{x + x_0}$ and remains constant transverse to the flow, while the macroscopic scale L_t decreases as $(x + x_0)^{-\frac{n+2}{4}}$. The characteristic ratio R of the time scales remains constant, with R = 0.6.

With $I_t \neq 0$, it follows unambiguously from (5) that $nT = -(n + 1)/2$, so $T(x, r) = T_0 (x + x_0)^{-\frac{n+1}{2}} \Phi(P_\infty)$; if there is no excess heat content, the integral condition does not enable us to determine nT. The solution to (5) can [6] be represented as $f_t = \Phi(P_\infty)\, {}_1F_1 \left(nT + \frac{n+1}{2}, \frac{n+1}{2}; \frac{P_\infty}{4} \eta^2\right)$, where ${}_1F_1(a, b, x) = \sum\limits_{m=0}^{\infty} \frac{(a)_m}{(b)_m} \frac{x^m}{m!}$ is a degenerate hypergeometric function. With $nT = -(n + 1)/2 - k$, where k is any positive integer, f_t satisfies $I_t = 0$ and decreases exponentially for $\eta \to \infty$. Each k corresponds to a particular solution $T_k = T_k^0 (x + x_0)^{-\frac{2k+n+1}{2}} \Phi(P_\infty)\, {}_1F_1\left(-k, \frac{n+1}{2}; \frac{P_\infty}{4} \eta^2\right)$. The general solution to (5) is written as $T = \sum\limits_{i=1}^{\infty} \alpha_i T_i$, and the temperature defect in the final stage will be described by the first term in the sum, which decreases more slowly than the others. Consequently, the asymptotic representation for T(x, r) is

$$T(x, r) = T_0 (x + x_0)^{-\frac{n+3}{2}} \Phi(P_\infty)\left(1 - \frac{P_\infty}{2}\frac{\eta^2}{n+1}\right).$$

If there is no excess heat content, the relative contribution from the generation to the balance equation for the second moments diminishes downstream in the strong-turbulence region, so we omit the term $E/3r\ \partial T/\partial r$ in (2) for $I_t = 0$. Then (10) is supplemented with $nRT > nE + nT$, which implies

$$-\frac{2n+9}{2} < nRT < -\frac{n+5}{2}. \tag{14}$$

We multiply (6) by η^{n+2} and integrate with respect to η to get

$$\left(nRT + \frac{n+3}{2} + \frac{3}{4}c_{ut}\right)\int_0^\infty \eta^{n+2} f_{Rt}(\eta)\, d\eta = 0. \tag{15}$$

As the integral relation $\int_0^\infty \eta^{n+2} f_{Rt}(\eta)\, d\eta = 0$ is not a conservation law, it is possible for (15) to be obeyed only if $nRT = -(n + 3)/2 - 3/4\ c_{ut}$, so c_{ut} can take values in the range $4/3 < c_{ut} < 2(n + 6)/3$. As c_{ut} should be independent of the flow geometry, we have finally that

$$\frac{4}{3} < c_{ut} < 4;\ nRT = -\frac{n+3}{2} - \frac{3}{4}c_{ut}. \tag{16}$$

From (16) and (2) we see for the case $I_t \neq 0$ that the assumption that $E/3r\ \partial T/\partial r$ is small is correct only for $c_{ut} < 2(n + 4)/3$, so if the excess heat content is different from zero

$$nRT = \begin{cases} -\frac{n+3}{2} - \frac{3}{4}c_{ut}, & \text{if } \frac{4}{3} < c_{ut} < \frac{2(n+4)}{3}, \\ -\frac{2n+7}{3}, & \text{if } \frac{2(n+4)}{3} \leqslant c_{ut} < 4. \end{cases}$$

For example, in [4] it was assumed that $c_{ut} = 2$. Then the asymptotic representation for $R_t(x, r)$ is as follows, no matter what the excess heat content:

$$R_t(x, r) = R_{t0}(x + x_0)^{-\frac{n+6}{2}} \Phi(P_\infty).$$

These asymptotic representations may be useful in simulating wake turbulence throughout the range in the turbulent Reynolds and Peclet numbers. The numerical solution attains these asymptotes as R_λ and P_λ decrease, and insofar as this occurs, it indicates that the method of numerical integration is correct and that there are no errors in the program.

NOTATION

u_i and t, velocity and temperature fluctuations; $\varepsilon_u = \nu\overline{(\partial u_i/\partial x_k)^2}$, turbulent kinetic energy dissipation rate; $\varepsilon_t = \kappa\overline{(\partial t/\partial x_k)^2}$, spreading rate for scalar pulsations; $\lambda_u = \sqrt{5\nu q^2/\varepsilon_u}$, $\lambda_t = \sqrt{6\kappa \overline{t^2}/\varepsilon_t}$, microscales for vector and scalar fields; $q^2 = \overline{u_i u_i}$, doubled velocity-fluctuation kinetic energy; $L_u = 5q^3/\varepsilon_u$, $L_t = 6q\overline{t^2}/\varepsilon_t$, macroscales for vector and scalar fields; $R_\lambda = q\lambda_u/\nu$, $P_\lambda = q\lambda_t/\kappa$, turbulent Reynolds and Peclet numbers.

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